UNIT-I :

→ Development
→ Definition
→ Characteristics & phases
→ Types of OR models
→ Applications
→ Linear Programming Problem (LPP):-
  → Formation
  → Simplex method
  → Artificial variable Techniques
    → BIG-M method
    → TWO-phase method
  → Graphical solution or method
  → Concept of Duality

*Operation Research:- OR focuses on the mathematical scoring of consequences of a decission making to ~~pt~~ optimize the use of time, effort, and Resources. The concept of OR is also used to avoid blunders.

→ Optimization:- The ~~act~~ act of obtaining best result (or) solution under any given circumstances is known as optimization.

→ Development of OR:-

- During the world war-2, scientists of united kingdom studied at the stratagic and practical problems and they also studied air and land defence problems.
- The aim of this study was to determine the effective utilization of limited military resources to win the battle.
- The technique was named as operation research.
- After world war-2 OR techniques were develop and deployed In decission making problems, In complicated problems, In various fields such as Industries, academics & many Government organizations.

→ Definition of OR:-

According to "church men", "Ackoff" and "Arnoff" defined OR as the application of scientific methods, techniques and tools to operation of a system with optimum solutions to the problems. Where as, optimum refers to the best possible alternative.

→ Characteristics:-

- OR makes use of scientific methods to solve the problems.
- O.R increases effectiveness of the management decission making ability.
- O.R make the use of computers to solve large and complex problems.
- O.R offers a quantitative solutions.
- O.R also taken in to account the Human factors.
- O.R ~~...~~ imbibes an inter disciplenary team approach.

14/06/19 → O.R Phases:-

The scientific method in O.R steady Generally involves the following three phases.

Phase I :- Judgmental phase
Phase-II :- Research phase.
Phase-III :- Action phase.

-> Flow Diagram

Judgmental phase

⇓

Research phase

⇓

Action phase

• ***Judgmental phase*:-**

This phase includes the following activities.

(i). Determination of the operations.

(ii). Establishment of the Objectivies & values related to the operations.

(iii). Determination of the suitable methods.

(iv) ~~co-ordination~~ formulation of the problems relative to the objective

• ***Research Phase*:-**

This phase utilizes for the following methodologies.

(i). Operations and data collections for a better under standing of the ~~bl~~ problems.

(ii). Formulation of hypothesis and models.

(iii). Observations and experimentations to test the hypothesis on basis of additional data.

(iv). Analysis of a available information & Verification of the hypothesis using the established ~~method~~ measure of effectiveness.

~~FR~~ Prediction of various results & consideration of alternative methods.

• ***Action phase*:-**

This phase involves making recommendations of the decision process. The recommendations can be made by those who identified and presented the problems (or) anyone who influances in which problems has occured.

-> ***Types of Model*:-**

*Model:- A model is idalized representation of a real life system. the objective of a model is to identify significant factors that effects the real life systems and their inter relation ships.

→ Physical Modal:- It includes all forms of diagram, graph and jobs. Theya designed to tackle specific problems. They bring out significant factors and inter relationships in Pictorial forms to facilitate analysis. they are two types of Physical Models.

(i) Iconic (ii) Anolog

(i) Iconic models:- These are images of object (or) system it represents on a smaller scale.

(ii) Analog models:- These are small physical systems having characteristic similar to the objects.

Ex: toys

→ Mathematical models:-

set of mathematical symbols to represents the ~~efficient~~ decision variables of the system. & the variables are interelated by mathematical ~~system~~ symbols.

some examples of this are Allocations, sequencing and replacement problems.

→ Models by Nature of Environment:-

This model can be further classified into two models

(i) Deterministic (ii) Probabilistic

Deterministic model:- In which every thing is described and results are defined such as EOP models

(ii) Probabilistic Models:- In which the input and output ~~variab~~ variables follow a defined probability distribution such as game theory, queing theory

→ Models by Extend of Generality:-

This model can be further classified into two types (i) General (ii) specific

(i) General models:- The models which you can apply in General to any problem.

Ex:- LPP

(ii) specific models:- specific models are the other hand that you can apply only under specific conditions. For example:- You can use sales responsing curve as a function of only in the market function

→ Applications of O.R:-

- Any problem it may be simple (or) complecated can use OR techniques to find the best possible solution
- The scope of OR will explain by seeing its applications in various fields of every day life.
  - Defence
  - In industries
  - planning
  - Agriculture
  - Hospitals
  - Transport sector.
  - Research and development.

*In Defence Operation:- In modern war fare the defence operations are carried out by 3 major independent Components namely.

Airforce, Army and Navy. The activities in each of these components can be further divided in 4 sub-components namely Administration, Intelligence operation and training & supply.

* ~~EA~~ The applications of modern war technology in each of the components of military organizations require knowledge in respective fields.

so, operation research will give expertise knowledge for solving complex, stratagic and technical problems.

* In Industry:- The business environments is always changing and any decision useful one time may not be so good some time later. so, we have to take new decisions according to our constraints with the help of operation research.

* Planning:- In modern times it has become necessary for every government to have carefully planning for economic development of country. O.R techniques can be fruitfully applied to maximise the capital income with minimum sacrifise and time.

Agriculture:- with increase in populalation there is a need to increase the agriculture output, but were not in a position to increase agriculture output due to lack of cultivation land and water, these are several restrictions Hence ~~these are~~ there is need to determine a cause of action serving the best from the given restrictions.

These problems can solve successfully with the help of the O.R techniques.

* Hospitals:- O.R methods can solve waiting problems in out patient's department of a big hospitals and administrative problems of the hospital organizations.

* Transportation:- we can apply different O.R methods to regulate the arrival of trains and processing. time minimize the passenger waiting time, ~~by~~ formulate suitable transportation policy. There by reducing the cost and time of transportation.

* Research and Development:-

we can apply O.R methologies in the field of R&D for several purposses such as to control and plan product ~~in lots~~ Introductions

19/6/19 → **Linear programming problem (LPP):**

LPP is a problem which involves a minimization or maximization of an objective function which is subjection to no.of linear constrains. The constrains could be equalities (or) inequalities.

LPP formulation:-

P5:- In a sweet shop, they used to make two differant types A & B. They used two different rawmaterials such as milk and sugar to make sweets. to make one packet of sweet 'A', they require 3 litres of milk & 3 kg of sugar. To make one packet of sweet 'B', they require two litres of milk & 4 kg's of sugar. they have 21 litres of milk & 28 kg's of sugar. Formulate a LPP to maximize the revenue.

Note:- The cost of one packet sweet 'A' is 1000/-
& the " " sweet 'B' is 900/-

Sol:- Let, 'x_1' be no.of sweet 'A' packets to be made
'x_2' be no.of sweet 'B' packets to be made

when we made 'x_1' no.of sweet 'A' packets and 'x_2' No.of sweet 'B' packets.

$\therefore$ Total Revenue $= 1000x_1 + 900x_2$

The objective of given data is to maximize

$$\boxed{\text{maximize } (Z) = 1000x_1 + 900x_2} \leftarrow \text{objective function}$$

To produce 'x_1' no.of sweet 'A' & 'x_2' no.of sweet 'B' packets, requires milk and sugar. The constrained eqns for milk & sugar is as follows.

$$3x_1 + 2x_2 \leq 21 \longrightarrow \text{milk constrained eqn.}$$
$$3x_1 + 4x_2 \leq 28 \longrightarrow \text{sugar constrained eqn}$$

So, the final LPP problem is as followes.

$$\text{Max } (Z) = 1000x_1 + 900x_2$$

subjected to

$$3x_1 + 2x_2 \leq 21$$
$$3x_1 + 4x_2 \leq 28$$
$$\therefore x_1, x_2 \geq 0.$$

20/6/19

P6:- A company is manufacturing two differant types of products 'A' & 'B'. each product has to be processed ~~on each com~~. two machines M_1 & M_2. Product 'A' requires 2 hours on machine M_1 and one hour on machine 'M_2'. & product 'B' requires one hour on M_1 and two hours on M_2. The avilable capacity of M_1 is 104 hours & M_2 is 76 hours. profit per unit for product 'A' is 6/- & that for 'B' is 11/-. calculate (i) formulate the problem (ii) find out optimum solution by simplex method.

Sol:

	A	B	total time
M_1	2	1	104
M_2	1	2	76

Profit for Product 'A' = 6/-
" product 'B' = 11/-

(i). Max $(z) = 6x_1 + 11x_2$

sub to $2x_1 + x_2 \leq 104$

$x_1 + 2x_2 \leq 76$

$x_1, x_2 \geq 0$

(ii). Simplex method:-

$\text{Max}(z) = 6x_1 + 11x_2 + 0S_1 + 0S_2$

sub to $2x_1 + x_2 + S_1 = 104 \rightarrow ①$

$x_1 + 2x_2 + S_2 = 76 \rightarrow ②$

Initial basic feasible solution [IBFS]

Put $x_1 = x_2 = 0$

$S_1 = 104$

$S_2 = 76$

optimality condition

$\boxed{C_j - Z_j < 0}$

	C_j	6	11	0	0		
C_B	B.V	x_1	x_2	S_1	S_2	Solution [IBFS]	min Ratio
0	S_1	2	1 key column	1	0 key row	104	$\frac{104}{1} = 104$
0	S_2	1	2 key element	0	1	76	$\frac{76}{2} = 38$
	Z_j	0	0	0	0	0	
	$C_j - Z_j$	6	11 ↑	0	0		

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$\therefore$ Final solution:

$$Max(Z) = 6x_1 + 11x_2 + 0s_1 + 0s_2$$
$$= (6\times 44) + (11\times 10) + (0\times 0) + (0\times 0)$$

$$\boxed{Max(Z) = 440/-}$$

2/6/19 (P6). Old hen's canbe bought at 30/- each and young one sold 50/- each. The old hens lay 3 eggs per week and the young ones lay 6 eggs per week, each egg being worth 1.75/- ~~paise~~. A hen (young (or) old) costs 3/- per week to feed. I have only 100/- to spend for hens. How many of each kind should I buy to give a profit of more than 6 per week, assuming that I cannot house more than 20 hens?

Sol:- x_1 — No. of old hens
x_2 — No. of young hens.

old hen cost — Rs 30/-
Young hen cost — Rs 50/-
Total amount — Rs 100/-

$$30x_1 + 50x_2 \le 100$$
$$x_1 + x_2 \le 20$$

w.k.t,
$\therefore$ Profit = Total amount — Expenditure

$$\text{Proft} = [(3x_1 + 6x_2)\times 1.75] - [3x_1 + 3x_2]$$
$$= [5.25x_1 + 10.5x_2] - [3x_1 + 3x_2]$$
$$= [5.25x_1 - 3x_1] + [10.5x_2 - 3x_2]$$
$$= 2.25x_1 + 7.5x_2$$

$$\therefore Max(Z) = 2.25x_1 + 7.5x_2$$
sub to,
$$30x_1 + 50x_2 \le 100$$
$$x_1 + x_2 \le 20$$
$$x_1, x_2 \ge 0$$

Simplex method:-

$$Max(Z) = 2.25x_1 + 7.5x_2 + 0s_1 + 0s_2$$
sub to $30x_1 + 50x_2 + s_1 = 100 \rightarrow (1)$
$x_1 + x_2 + s_2 = 20 \rightarrow (2)$

(P5):-

Q.B:2

Home work

$$\boxed{\begin{aligned} Max(z) &= -1000x_1 + 900x_2 \\ \text{sub to,} \quad & 3x_1 + 2x_2 \le 21 \\ & 2x_1 + 4x_2 \le 28 \\ & x_1, x_2 \ge 0 \end{aligned}}$$

Initial basic feasible solution (IBFS):-

Put, $x_1 = x_2 = 0$

$S_1 = 100$

$S_2 = 20$

	C_j	2.25	7.5	0	0		
C_B	B.V.	x_1	x_2	S_1	S_2	solution	Min ratio
0	S_1	30	50	1	0	100	$\frac{100}{50} = 2$
0	S_2	1	1	0	1	20	$\frac{20}{1} = 20$
	Z_j	0	0	0	0		
	$C_j - Z_j$	2.25	7.5	0	0		

key element

key column

key row

	C_j	2.25	7.5	0	0	
C_B	B.V	x_1	x_2	S_1	S_2	solution
7.5	x_2	$\frac{3}{5}$	1	$\frac{1}{50}$	0	$\frac{100}{2}$ 2
0	S_2	1	1	0	1	20

$R_2 \to R_2 - R_1$

	C_j	2.25	7.5	0	0	
C_B	B.V	x_1	x_2	S_1	S_2	solution
7.5	x_2	$\frac{3}{5}$	1	$\frac{1}{50}$	0	2
0	S_2	$\frac{2}{3}$	0	$-\frac{1}{50}$	1	18
	Z_j	4.5	7.5	0.15	0	
	$C_j - Z_j$	-2.25	0	-0.15	0	

Now, $x_2 = 2$, $S_2 = 18$ | $x_1 = 0$, $S_1 = 0$

$\therefore$ Final solution is

$$Max(z) = 2.25x_1 + 7.5x_2 + 0s_1 + 0s_2$$

$$= 2.25(0) + 7.5(2) + (0 \cdot 0) + (0 \times 18)$$

$$\boxed{Max(z) = 15}$$

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P6:-

Q.B:6

$$Min(z) = x_1 - 3x_2 + 2x_3$$

sub to

$$3x_1 - x_2 + 3x_3 \le 7$$

$$-2x_1 + 4x_2 \le 12$$

$$-4x_1 + 3x_2 + 8x_3 \le 10$$

where, $x_1, x_2, x_3 \ge 0$

Sol:- The given function is in minimization manner. So we can convert into Maximization function.

$$Max(z) = Min(-z) = -(x_1 - 3x_2 + 2x_3)$$

$$= -x_1 + 3x_2 - 2x_3$$

sub.to

$$3x_1 - x_2 + 3x_3 \le 7$$

$$-2x_1 + 4x_2 \le 12$$

$$-4x_1 + 3x_2 + 8x_3 \le 10$$

$$x_1, x_2, x_3 \ge 0$$

Max (Z) = $-x_1 + 3x_2 - 2x_3 + 0S_1 + 0S_2 + 0S_3$

sub to,

$3x_1 - x_2 + 3x_3 + S_1 = 7 \rightarrow (1)$

$-2x_1 + 4x_2 + S_2 = 12 \rightarrow (2)$

$-4x_1 + 3x_2 + 8x_3 + S_3 = 10 \rightarrow (3)$

Initial basic feasible solution (IBFS):- for given problem

Put, $x_1 = x_2 = x_3 = 0$

$S_1 = 7$

$S_2 = 12$

$S_3 = 10$.

	C_j	-1	3	-2	0	0	0		
CB	B.V	x_1	x_2	x_3	S_1	S_2	S_3	Solution (IBFS)	min ratio
0	S_1	3	-1	3	1	0	0	7	–
0	S_2	-2	4	0	0	1	0	12	3
0	S_3	-4	3	8	0	0	1	10	3.33
	Z_j	0	0	0	0	0	0		
	$C_j - Z_j$	-1	3	-2	0	0	0		

↑

$R_2 \rightarrow R_2/4$

	C_j	-1	3	-2	0	0	0	
CB	B.V	x_1	x_2	x_3	S_1	S_2	S_3	solution
0	S_1	3	-1	3	1	0	0	7
3	x_2	$-\frac{1}{2}$	1	0	0	$\frac{1}{4}$	0	3
0	S_3	-4	3	8	0	0	1	10

$R_1 \rightarrow R_1 + R_2$

$R_3 \rightarrow R_3 - 3R_2$

	C_j	-1	3	-2	0	0	0		
CB	B.V	x_1	x_2	x_3	S_1	S_2	S_3	solution	min ratio
0	S_1	$\frac{5}{2}$	0	3	1	$\frac{1}{4}$	0	10	4
3	x_2	$-\frac{1}{2}$	1	0	0	$\frac{1}{4}$	0	3	–
0	S_3	$-\frac{5}{2}$	0	8	0	$\frac{-3}{4}$	0	1	–
	Z_j	$-\frac{3}{2}$	3	0	0	$\frac{3}{4}$	0		
	$C_j - Z_j$	$+\frac{1}{2}$	0	-2	0	$-\frac{3}{4}$	0		

↑

$R_1 \rightarrow R_1 \times \frac{2}{5}$

	C_j	-1	3	-2	0	0	0	
CB	B.V	x_1	x_2	x_3	S_1	S_2	S_3	solution
-1	x_1	1	0	$\frac{6}{5}$	$\frac{2}{5}$	$\frac{1}{10}$	0	$10(\frac{2}{5}) = 4$
3	x_2	$-\frac{1}{2}$	1	0	0	$\frac{1}{4}$	0	3
0	S_3	$-\frac{5}{2}$	0	8	0	$\frac{-3}{4}$	1	1

$R_2 \rightarrow R_2 + \frac{R_1}{2}$

$R_3 \rightarrow R_3 + \frac{5R_1}{2}$

	C_j	-1	3	-2	0	0	0	
CB	B.V	x_1	x_2	x_3	S_1	S_2	S_3	solution
-1	x_1	1	0	$\frac{6}{5}$	$\frac{2}{5}$	$\frac{1}{10}$	0	4
3	x_2	0	1	$\frac{6}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	0	5
0	S_3	0	0	11	1	$-\frac{1}{2}$	1	11
	Z_j	-1	3	$+\frac{9}{5}$	$\frac{1}{5}$	$\frac{4}{5}$	0	
	$C_j - Z_j$	0	0	$\frac{-13}{5}$	$-\frac{1}{5}$	$-\frac{4}{5}$	0	

$x_1 = 4$; $x_2 = 5$; $S_3 = 11$

Max (Z) $= -x_1 + 3x_2 - 2x_3 + 0S_1 + 0S_2 + 0S_3$

$= -4 + 3(5) - 2(0) + 0(0) + 0(0) + 0(11)$

$= -4 + 15$

$\boxed{\text{Max (Z)} = 11}$

25/06/19 → Simplex algorithm:-

1). Formulate the problem. If the given objective function is minimization type, we need to convert it into maximization type by simply multiplying with negative sign.

$$\boxed{\text{Max}(z) = \min(-z).}$$

2). Convert all inequalities in the constrained equations into equalities by introducing slack (or) surplus variables on the left hand side.

for the General linear programming problem we can rewrite the constraints by adding slack variables

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n + s_1 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n + s_2 = b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n + s_m = b_m$$

3) Find Initial basic feasible solution for the given problem, for finding Initial basic feasible solution put "Zero" value for decission variables so, IBFS for the Generalized LPP is as followes.

$\therefore$ $s_1 = b_1$
$s_2 = b_2$
$\vdots$
$s_m = b_m$

4). The initial simplex table is framed by writing all the coefficients and constraints in a systamatic tabular form.

	c_j	c_1	c_2	c_3	$\cdots$	c_n	Solution
C_B	B.V	x_1	x_2	x_3	$\cdots$	x_n	
C_{B1}	s_1	a_{11}	a_{12}	a_{13}	$\cdots$	a_{1n}	b_1
C_{B2}	s_2	a_{21}	a_{22}	a_{23}	$\cdots$	a_{2n}	b_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
C_{Bm}	s_n	a_{m1}	a_{m2}	a_{m3}	$\cdots$	a_{mn}	b_m
	z_j	z_1	z_2	z_3	$\cdots$	z_n	

~~calculation of Zj values~~.

calculation of Zj values:-

$$z_1 = C_{B1} \times a_{11} + C_{B2} \times a_{21} + \cdots C_{Bn} \times a_{m1}$$
$$z_2 = C_{B1} \times a_{12} + C_{B2} \times a_{22} + \cdots C_{Bn} \times a_{m2}$$
$$z_3 = C_{B1} \times a_{13} + C_{B2} \times a_{23} + \cdots C_{Bn} \times a_{m3}$$
$$\vdots$$
$$z_n = C_{B1} \times a_{1n} + C_{B2} \times a_{2n} + \cdots C_{Bn} \times a_{mn}$$

After finding Zj values.

5). Check for the optimality condition to the given problem.

If the problem is maximization type, we need to verify "$c_j - z_j$" values.

If "$c_j - z_j$" values value is less than (or) equal to "Zero" so we reach optimum solution for the given problem.

If "$c_j - z_j$" values is not equal to zero (or not less than zero the problem is not reach optimal solution so for reaching optimal solution for the given problem we need to move to next iteration.

In view of developing next iteration we need to find key column, key row and key element. After that a decission variable is introduced inplace of IBFS.

6). Again we will check optimal condition for the present iteration. If "$c_j - z_j$" values is less than (or equal to zero so the problem is reaches optimal solution. else we need to move further iteration, the same procedure continued till the problem reaches optimal condition.

2/6/18 → Artificial Variable Technique :-

① 2- Phase method

② Big m-method.

Pb:- Use two phase method to solve the following LPP.

Q.B:-10

$$Max(z) = 2x_1 + x_2 + x_3$$

subject to,

$$4x_1 + 6x_2 + 3x_3 \le 8$$
$$3x_1 - 6x_2 - 4x_3 \le 1$$
$$2x_1 + 3x_2 - 5x_3 \ge 4$$
$$x_1, x_2, x_3 \ge 0$$

Sol:-

$$Max(z) = 2x_1 + x_2 + x_3$$

sub to,

$$4x_1 + 6x_2 + 3x_3 + s_1 = 8$$
$$3x_1 - 6x_2 - 4x_3 + s_2 = 1$$
$$2x_1 + 3x_2 - 5x_3 - s_3 + A_1 = 4$$
$$x_1, x_2, x_3, s_1, s_2, s_3 \ge 0$$

Initial basic feasible solution (IBFS):-

$$x_1 = x_2 = x_3 = 0$$
$$s_1 = 8$$
$$s_2 = 1$$
$$A_1 = 4$$

Phase-I:-

$$Max(z) = 0x_1 + 0x_2 + 0x_3 + 0s_1 + 0s_2 + 0s_3 - A_1$$

sub.to,

$$4x_1 + 6x_2 + 3x_3 + s_1 = 8$$
$$3x_1 - 6x_2 - 4x_3 + s_1 = 1$$
$$2x_1 - 3x_2 - 5x_3 - s_3 + A_1 = 4$$
$$x_1, x_2, x_3, s_1, s_2, s_3, A_1 \ge 0$$

C_B	C_j	0	0	0	0	0	0	-1		
	BV	x_1	x_2	x_3	s_1	s_2	s_3	A_1	solution	Min ratio
0	s_1	4	6	3	1	0	0	0	8	1.33
0	s_2	3	-6	-4	0	1	0	0	1	—
-1	A_1	2	3	-5	0	0	-1	1	1	
	z_j	-2	-3	5	0	0	1	-1	4	1.33
	$c_j - z_j$	2	3 ↑	-5	0	0	-1	0		

C_B	C_j	0	0	0	0	0	0	
	BV	x_1	x_2	x_3	s_1	s_2	s_3	solution
0	s_1	4	6	3	1	0	0	8
0	s_2	3	-6	-4	0	1	0	1
0	x_2	2	3	-5	0	0	-1	4

Phase-II:

$$Max(z) = 2x_1 + x_2 + x_3 + 0s_1 + 0s_2 + 0s_3.$$

	C_j	2	1	1	0	0	0	
C_B	BV	x_1	x_2	x_3	s_1	s_2	s_3	solution
0	s_1	4	6	3	1	0	0	8
0	s_2	3	-6	-4	0	1	0	1
+1	x_2	2	3	-5	0	0	-1	4

	c_j	2	1	1	0	0	0		
CB	BV	x_1	x_2	x_3	s_1	s_2	s_3	solution	Min ratio
0	s_1	0	0	3	1	0	2	0	← 0
0	s_2	-1	0	-4	0	1	-2	9	
-1	x_2	$\frac{2}{3}$	1	$-\frac{5}{3}$	0	0	$-\frac{1}{3}$	$\frac{4}{3}$	
	z_j	$\frac{2}{3}$	+1	$-\frac{5}{3}$	0	0	$-\frac{1}{3}$		
	$c_j - z_j$	$\frac{4}{3}$	0	$\frac{8}{3}$ ↑	0	0	$\frac{1}{3}$		

$R_3 \rightarrow \frac{R_3}{3}$

$R_1 \rightarrow R_1 - [illegible]$

$R_2 \rightarrow R_2 + [illegible]$

Phase - II

	C_j	-4	-3	-1	0	0	
C_B	B.V	x_1	x_2	x_3	S_1	S_2	sol.
-1	x_3	$\frac{1}{4}$	$\frac{1}{2}$	1	$\frac{-1}{4}$	0	3
-4	x_1	$\frac{11}{4}$	$\frac{3}{2}$	0	$\frac{1}{4}$	-1	5

~~Z_j $\frac{-45}{4}$ $\frac{-13}{2}$ -1 $\frac{-3}{4}$ 4~~

~~$C_j - Z_j$ $\frac{29}{4}$~~

$R_2 \to R_2 \div \frac{11}{4}$; $R_1 \to R_1 - \frac{R_2}{4}$

	C_j	-4	-3	-1	0	0		
C_B	B.V	x_1	x_2	x_3	S_1	S_2	sol	min ratio
-1	x_3	0	$\frac{4}{11}$	1	$\frac{-3}{11}$	$\frac{1}{11}$	$\frac{28}{11}$	—
-4	x_1	1	$\frac{6}{11}$	0	$\frac{1}{11}$	$\frac{-4}{11}$	$\frac{20}{11}$	20 ←
	Z_j	-4	$\frac{-28}{11}$	-1	$\frac{-1}{11}$	$\frac{15}{11}$		
	$C_j - Z_j$	0	$\frac{-5}{11}$	0	$\frac{1}{11}$	$\frac{-15}{11}$		

↑

$R_1 \to R_1 + \frac{3}{11}R_2$ $R_2 \to 11R_2$

	C_j	-4	-3	-1	0	0	
C_B	B.V	x_1	x_2	x_3	S_1	S_2	sol
-1	x_3	3	2	1	0	-1	8
0	S_1	11	6	0	1	-4	20
	Z_j	-3	-2	-1	0	+1	
	$C_j - Z_j$	-1	-1	0	0	-1	

Big-M method:

Q8:-1:- P6!

$\text{Max}(z) = 3x_1 - x_2$

sub to, $2x_1 + x_2 \geq 2$

$x_1 + 3x_2 \leq 3$

$x_2 \leq 4$

$x_1, x_2 \geq 0$

$\text{Max}(z) = 3x_1 - x_2 + 0s_1 + 0s_2 + 0s_3 - MA_1$

sub to,

$2x_1 + x_2 - s_1 + A_1 = 2$

$x_1 + 3x_2 + s_2 = 3$

$0x_1 + x_2 + s_3 = 4$

$x_1, x_2, s_1, s_2, s_3, A_1 \geq 0$

	C_j	3	-1	0	0	0	-M		
C_B	BV	x_1	x_2	s_1	s_2	s_3	A_1	sol	Min ratio
-M	A_1	2	1	-1	0	0	~~1~~	2	$\frac{2}{2}=1$ ←
0	s_2	1	3	0	1	0	0	3	$\frac{3}{1}=3$
0	s_3	0	1	0	0	1	0	4	∞
	z_j	-2M	-M	M	0	0	-M		
	$C_j - z_j$	3+2M ↑	-1+M	-M	0	0	0		

$R_1 \rightarrow R_1/2 \; ; \; R_2 \rightarrow R_2 - R_1$

	C_j	3	-1	0	0	0		
C_B	B.V	x_1	x_2	s_1	s_2	s_3	sol	min ratio
3	x_1	1	$\frac{1}{2}$	$\frac{-1}{2}$	0	0	1	—
0	s_2	0	$\frac{5}{2}$	$\frac{1}{2}$	1	0	2	4 ←
0	s_3	0	1	0	0	1	4	∞
	z_j	3	$\frac{3}{2}$	$\frac{-3}{2}$	0	0		
	$C_j - z_j$	0	$-\frac{5}{2}$	$\frac{3}{2}$ ↑	0	0		

$R_2 \rightarrow R_2 \times 2$

$R_1 \rightarrow R_1 + \frac{1}{2}R_2$

	C_j	3	-1	0	0	0	
C_B	BV	x_1	x_2	s_1	s_2	s_3	sol
3	x_1	1	3	0	1	0	3
0	s_1	0	5	1	2	0	4
0	s_3	0	1	0	0	1	4
	z_j	3	9	0	3	0	
	$C_j - z_j$	0	-10	0	-3	0	

$\therefore C_j - z_j \leq 0$ hence optimality condition is reached

$\therefore x_1 = 3 \; ; \; s_1 = 4 \; ; \; s_3 = 4$

and $x_2 = 0 \; ; \; s_2 = 0$

$\therefore \text{Max}(z) = 3x_1 - x_2$

$= 3(3) - 0$

$\boxed{\text{Max}(z) = 9}$

2/7/18 → Concept of Duality :-

$$Max(Z) = 2x_1 + x_2$$

sub to,

$$x_1 + x_2 \le 8$$

$$2x_1 + x_2 \le 7$$

$$x_1 + 5x_2 \ge 3$$

$$2x_1 + 7x_2 \le 6$$

$$5x_1 + x_2 \le 3$$

$$6x_1 + 3x_2 \ge 6$$

$$x_1 + 5x_2 \ge 1 \quad \text{and } x_1, x_2 \ge 0$$

Sol:

$$Min(Z) = 8x_1 + 7x_2 + 3x_3 + 6x_4 + 3x_5 + 6x_6 + x_7$$

sub to,

$$x_1 + 2x_2 + x_3 + 2x_4 + 5x_5 + 6x_6 + x_7 \ge 2$$

$$x_1 + x_2 + 5x_3 + 7x_4 + x_5 + 3x_6 + 5x_7 \ge 1$$

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \ge 0$$

→ Graphical Soultion :-

*Note :- This is applical when Only TWO - Variable are present in the Constrained eqn.

Q8.1 Given, $Max(Z) = 3x_1 - x_2$

sub to, $2x_1 + x_2 \ge 2$

$$x_1 + 3x_2 \le 3$$

$$\cancel{x_1 + x_2 \ge 0}$$

$$x_2 \le 4$$

$$x_1, x_2 > 0.$$

(P8). A company produces 2 products A&B. ~~He~~

Two products A & B has to be processed on two machines M_1 & M_2. For product 'A' it requires 2 hour of processing time on M_1 & ones hour processing time on M_2, For product 'B' 1 hour on M_1 & 2 hours on M_2. The total Machined houres available on M_1 is 104 hours & on M_2 76 hours. The profit on the one unit of product 'A' is 6/- and on the product 'B' is 11/-. Formulate LPP & solve using graphical method.

Sol:

	A	B	total hours
M_1	2	1	104
M_2	1	2	76
profit	6/-	11/-	

If profits → we need to Maximize

If expenses → we need to minimize

x_1 → Quantity of product 'A' to be produced.

x_2 → Quantity of product 'B' to be produced.

$6x_1$ → $11x_2$.

Max $Z = \boxed{6x_1 + 11x_2}$ ← we need to Maximize this function.

subject to, $2x_1 + x_2 \le 104$ —(1)

$x_1 + 2x_2 \le 76$ —(2)

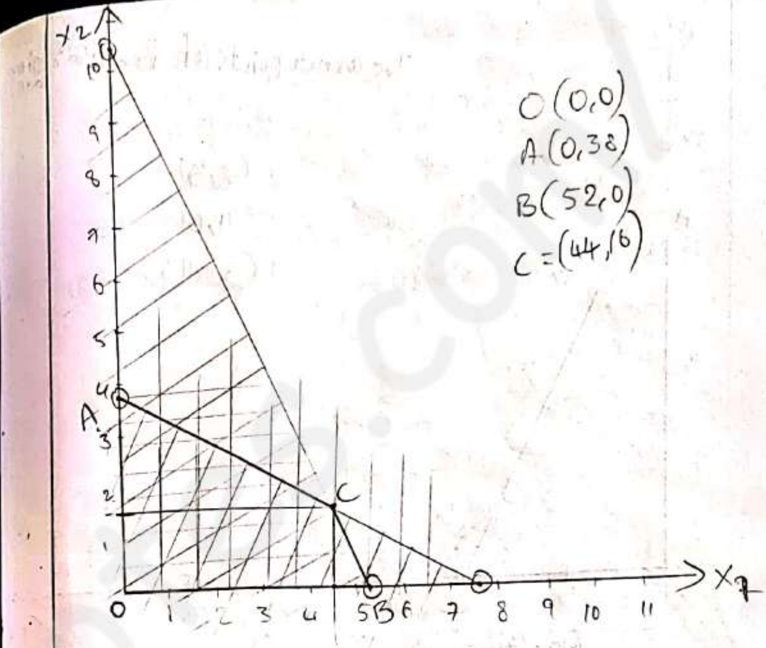
$x_1 \ge 0;\ x_2 \ge 0$

For eqn (1), $\Rightarrow \frac{x_1}{52} + \frac{x_2}{104} \le 1 \Rightarrow (52,0)$ & $(0,104)$

for eqn (2) $\Rightarrow \frac{x_1}{76} + \frac{x_2}{38} \le 1 \Rightarrow (76,0)$ & $(0,38)$

Take (5.2,0 & (0,104)

Take (7.6,0 & (0.3.8)



O(0,0)

A(0,38)

B(52,0)

C=(44,16)

$Max(Z) = 6x_1 + 11x_2$

$Max(Z)_O = 0$

$Max(Z)_A = 0 + 11 \times 38 = 418$

$Max(Z)_B = 6 \times 52 + 0 = 312$

$Max(Z)_C = 44 \times 6 + 16 \times 11 = 440$

Material PS: No-5 Example-1

$Max(Z) = 18x_1 + 16x_2$

subject to, $15x_1 + 25x_2 \le 375$ → (1)

$24x_1 + 11x_2 \le 264$ → (2)

$x_1 \ge 0$

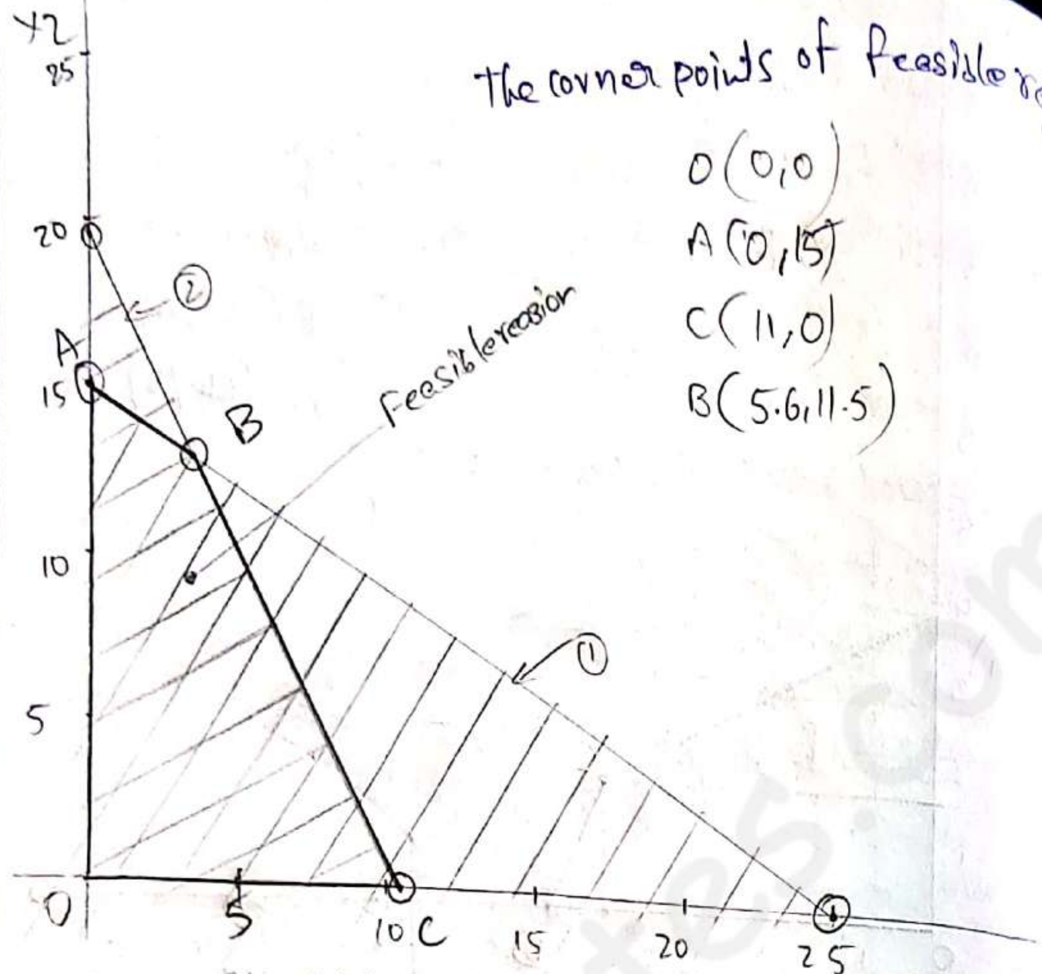
$x_2 \ge 0$.

for (1) $\Rightarrow \frac{x_1}{25} + \frac{x_2}{15} \le 1 \Rightarrow (25,0)$ & $(0,15)$

for (2) $\Rightarrow \frac{x_1}{11} + \frac{x_2}{24} \le 1 \Rightarrow (11,0)$ & $(0,24)$

Take.

Scanned by CamScanner



The corner points of feasible region are

$O(0,0)$

$A(0,15)$

$C(11,0)$

$B(5.6,11.5)$

$\therefore$ $\text{Max}(Z)_O = 0$

$\text{Max}(Z)_A = 18(0)+16(15) = 240$

$\text{Max}(Z)_B = 18(5.6)+16(11.5) = 284.8$

$\text{Max}(Z)_C = 18(11)+16(0) = 198$

$\therefore$ The optimal solution is at point the corner point B. and the optimal solution is $x_1 = 5.6$.

$x_2 = 11.5$.

Material PS:-7

(Pb). $Max(Z) = x_1 + 2x_2$ —①

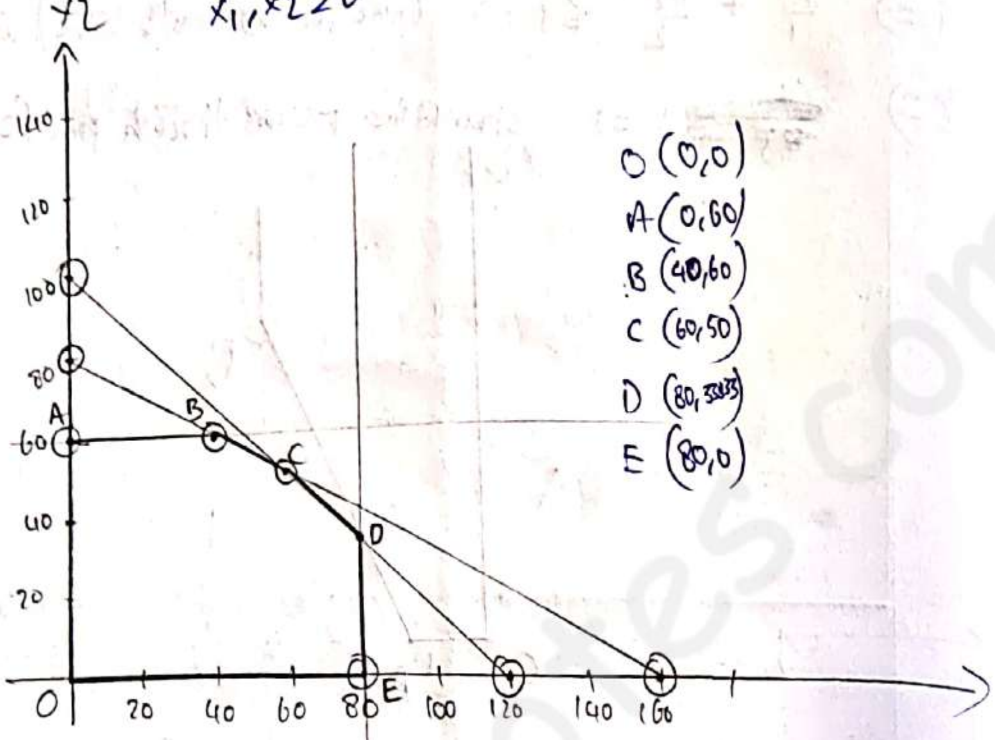
sub to $x_1 \le 80$ —②

$x_2 \le 60$ —③

$5x_1 + 6x_2 \le 600$ —④ $\Rightarrow \frac{x_1}{120} + \frac{x_2}{100} \le 1$ (120,0) (0,...)

$x_1 + 2x_2 \le 160$ —⑤ $\Rightarrow \frac{x_1}{160} + \frac{x_2}{80} \le 1$ (160,0) (0,...)

$x_1, x_2 \ge 0$.



To get 'B' we solve $x_2 = 60$ & ③

Then
$Z_O = 0$
$Z_A = 120$
$Z_B = 160$
$Z_C = 160$
$Z_D = 146.66$
$Z_E = 80$

The given linear programming problem is having two optimal solutions at point 'B' & 'C'. The optimal solutions are

① sol-1 : $x_1 = 40$; $x_2 = 60$

② sol-2 : $x_1 = 60$ & $x_2 = 50$

PS:- 8) $min(Z) = 200x_1 + 300x_2$

sub to, $2x_1 + 3x_2 \ge 1200$

$x_1 + x_2 \le 400$

$2x_1 + 1.5x_2 \ge 900$

$x_1, x_2 \ge 0$.

$\frac{x_1}{600} + \frac{x_2}{400} \ge 1 \Rightarrow$ line joining point are (600,0) ; (0,400)

$\frac{x_1}{400} + \frac{x_2}{400} \ge 1 \Rightarrow$ (400,0) & (0,400)

$\frac{x_1}{450} + \frac{x_2}{600} \ge 1 \Rightarrow$ (450,0) & (0,600).